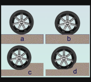
Mechanics of Wheeled Mobility on Hard Ground

You design for the worst case/terrain.

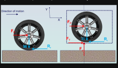
Conditions of wheel-terrain interaction:



(a) Rigid wheel on hard terrain
(b) Flexible/elastic wheel on hard terrain
(c) Rigid wheel on deformable terrain
(d) Flexible/elastic wheel on deformable terrain

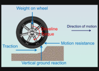
Free-body Diagram:

W = weight
Rr = resistance to motion / Rolling resistance
Fp = force that pushes or pulls the wheel along

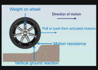


Sum of all forces and moments should be zero

Forces on powered/driven wheel:



Forces on towed/non-driven wheel:



Traction:

Frictional force that allows wheel to grip the surface its moving on, preventing it from slipping.
In context of wheeled locomotion, friction is dry friction. It is caused by interaction of surface microfeatures. It is part adhesion and part abrasion.

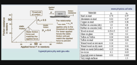
Static friction:
- Forces that resists the initiation of motion between two surfaces that are not moving relative to each other.
- Occurs when an object is stationary and you try to move it.
- Magnitude increases with the force applied up to a certain limit.

$$f_{static} \leq \mu_s \cdot W$$

Kinetic friction:
- Force that resists the motion of two surfaces sliding past each other once they are already moving.
- Acts when object is in motion
- Magnitude is usually lower than static friction.

$$f_{kinetic} = \mu_k \cdot W$$

Coefficients of friction -
- independent of shape and size of contact area
- depends on type of materials in contact
- independent of velocity at contact



Rolling resistance:

The resistance to motion. Caused by -

- Hysteresis in material:
  - Happens because the material of wheel deforms as it comes in contact with ground, and then recovers its shape after leaving contact.
  - When wheel deforms, energy is absorbed by the material. But not all this energy is returned back . This energy loss causes resistance.
  - When the wheel moves forward, new sections of the tire deform, and previous sections recover, causing a continuous cycle of energy loss.

- Sliding losses:
  - Even though the wheel primarily rolls, there is often a small amount of slipping or sliding between the tire and the surface, particularly at the contact point where the tire touches the ground.
  - This sliding causes friction, which leads to energy being converted into heat, further increasing rolling resistance.

- Air circulation:
  - Air trapped inside may move around as the tire deforms and rolls. This circulation of air inside the tire can create internal friction, leading to additional energy losses and, consequently, higher rolling resistance.

- Affected by type of material, tire design, speed, inflation pressure, operational conditions

$$R_r = f_R W$$

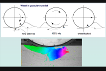
rolling resistance coefficient

car tires $f_r = 0.0136 + 0.4 \times 10^{-7} v^2$

truck tires $f_r = 0.006 + 0.23 \times 10^{-6} v^2$

Fundamentals of wheel-soil interaction

- The wheel sinks into the soil and compresses and displaces the soil mass in front and under it
- As the soil tries to resist to the rotation of the wheel, it experiences stresses (forces over unit area)
- Stresses parallel to wheel-soil interface produce traction known as soil thrust
- Phenomena depends on slip, loading, wheel/track geometry
- Energy is expanded in compacting and displacing the soil



Wheel-soil interface

Normal stress - relates to ground pressure and sinkage

$$\sigma = p = \frac{W}{A}$$

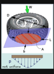
Shear stress - relates to traction, slip, and skid

$$\tau = \frac{F}{A}$$

Ground pressure - acts at contact patch

$$\sigma = p = \frac{W}{A}$$ contact patch

b = width of contact patch
L = length of contact patch



Strain $\varepsilon = \frac{\Delta l}{l}$

Condition for soil to fail

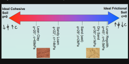
Mohr-Coulomb failure criterion: Material will fail if the shear stress at a point meets the following condition

$$\tau = c + \sigma \tan\phi$$

shear stress
soil angle of internal resistance to shearing
normal stress due to compression
soil cohesion

Cohesion: Relates to the shear strength that holds together soil particles (other than friction) and is independent of normal pressure.

Internal friction angle / Angle of internal resistance to shearing:
Angle at which the material will slip on its own surface



Soil Thrust (H) / Traction:
Max applied shear force that the soil will sustain before failing (breaking)
= max traction available

$$\tau = c + \sigma \tan\phi$$
$$\tau A = cA + \sigma A \tan\phi$$
$$H = cA + W \tan\phi$$

soil thrust is limited by wheel slip

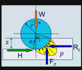
$$H = \underbrace{(cA + W\tan\phi)}_{\text{ideal soil thrust}} f(s)$$

slip term

soil thrust provides traction

Motion resistance due to soil compaction

In deformable terrains (like soft soil), the wheel or track sinks into the ground, and the soil gets compacted. The resistance to motion (denoted as R) is a result of this compaction.
We can model this motion resistance by integrating the pressure distribution over the contact area between the wheel and the soil.



$$R_c = b \int_0^{z_{max}} p(z)\, dz$$

Rc = motion resistance due to soil compression
b = width of wheel
p(z) = pressure at depth z in soil
z = depth of soil sinkage

Bekker's pressure sinkage semi-empirical equation

$$p(z) = k z^n$$

k,n = soil parameters
z = sinkage

cohesion coefficient

$$p = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

sinkage exponent
width of wheel
Angle of internal friction coefficient

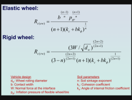
$$R_c = b \int_0^{z_{max}} p\, dz = b \int_0^{z_{max}} \left(\frac{k_c}{b} + k_\phi\right) z^n\, dz$$

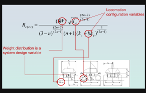
For flexible wheel:

$$z_{rw} = \left[\frac{p}{\left(\frac{k_c}{b} + k_\phi\right)}\right]^{1/n}$$

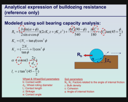
For rigid wheel:

$$z_{rw} = \left[\frac{3W}{(3-n)(k_c + k_\phi b)\sqrt{d}}\right]^{\frac{2}{2n+1}}$$





Motion resistance due to bulldozing



Drawbar Pull:
Max horizontal pulling force that a vehicle can exert at the drawbar (connection point where the vehicle pulls an implement or trailer) while maintaining forward motion.
It is a measure of how much net traction is available after resistances are overcome and before the soil fails in shear.

DP = H - sum(R)

max traversable slope θ_{max}

$$\tan\theta = \frac{DP}{F_z}$$

sum(R) = R compaction + R bulldozing + R gravitational + R obstacles + R turning + R internal

resistance due to engagement with granular material

Drive torque at wheel output to overcome motion resistance

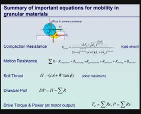
$$T_d = \sum R r$$

Drive power at wheel output to overcome motion resistance

$$P_d = \sum R v = \sum R \omega r$$

r = rolling radius of wheel
v = speed
w = angular velocity of wheel

SUMMARY:



Metrics for Evaluation

Mean Maximum Pressure (MMP)

$$MMP = \frac{6.895\, W_G}{n a b^{0.9} d^{0.9} \delta^{0.4}}$$

quantifies the average peak pressure exerted on a surface (such as soil or pavement) by a load

WG = gross vehicle weight (kN)
n = number of wheels per axle
b = average unloaded tire width (m)
d = average unloaded tire diameter (m)
δ = average tire deflection (how much the tire compresses under load) (m)
a = total number of axles

Higher pressures can lead to more significant soil compaction and deeper sinkage, which impacts vehicle mobility.

Vehicle Cone Index (VCI)

Reflects degree of floatation and traction achievable for a given vehicle on a given soil

VCI is the minimum value of soil strength at which a vehicle can successfully complete one pass, given that the vehicle is moving on level ground at a slow, steady speed and not pushing or towing